

Bachelor / Master Thesis

The game chromatic index of forests

We consider the following edge coloring game on a given graph G : For a given set of k colors, Maker and Breaker take turns in which they select an uncolored edge of the graph and assign a color such that two adjacent edges never get the same color. Maker wins if at the end, the whole graph is colored, and Breaker wins if at some point in time there exists an edge where every color is forbidden. We are interested to determine the smallest number $\chi'_g(G)$ such that Maker has a winning strategy for the game with $\chi'_g(G)$ colors.

Clearly, for every graph G of maximum degree $\Delta = \Delta(G)$ it holds $\Delta \leq \chi'_g(G) \leq 2\Delta - 1$. In this thesis, we would consider the game played on forests. It is not difficult to find a forest T of maximum degree Δ and a winning-strategy for Breaker, given that the number of colors is only Δ . Thus for every forest T we have $\chi'_g(T) \geq \Delta + 1$. On the other hand, if $k = \Delta + 2$ there exists a winning-strategy for Maker, which actually works for a more general class of graphs [1].

Some years ago, Faigle et al. [3] proved that for all forests of maximum degree at least 6, the lower bound of $\Delta + 1$ is indeed correct and $\Delta + 1$ colors suffice for Maker to win the game. Hence, only the cases of small maximum degree are open. For $\Delta = 2$, it is trivial to see that $\chi'_g(T) \leq 3$ (of course, if at least one path of the forest is long enough). For $\Delta = 3$, $\chi'_g(T) \leq 4$ is also proven by several case distinctions. The case $\Delta = 5$ was solved by Andres [2] by refining the strategy of [3].

We see that only the case $\Delta = 4$ is open. Your job would be to determine for forests of maximum degree 4 whether $\chi'_g(T) \leq 5$ is true, or finding a tree T such that $\chi'_g(T) = 6$. Andres claims that his strategy can not be used for this special case, and one needs to come up with new ideas. This would then be the topic of your thesis.

Prerequisites: Attended and passed the lecture Randomized Algorithms and Probabilistic Methods. In case of Bachelor thesis, APC is fine as well.

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[1] L. Cai, X. Zhu, *Game chromatic index of k -degenerate graphs*, Journal of Graph Theory 36 (2001), pages 144-155.

[2] S. Andres, *The game chromatic index of forests of maximum degree $\Delta \geq 5$* , Discrete Applied Mathematics 154 (2006), pages 1317-1323.

[3] P. Erdős, U. Faigle, W. Hochstättler, W. Kern, *Note on the game chromatic index of trees*, Theoretical Computer Science 313 (2004), pages 371-376.